ComPlexUs
REVIEW
Complexus 2003;1:190–206; DOI: 10.1159/000082449
Received: August 8, 2004
Accepted after revision: September 13, 2004
Published online: November 26, 2004

The Physical Modelling of Human Social Systems

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Nature, London, UK

For editorial comment see p. 207

Key Words
Sociology · Phase transitions · Statistical physics · History of science · Collective behaviour

Abstract
One of the core assumptions in the study of complex systems is that there exist 'universal' features analogous to those that characterize the notion of universality in statistical physics. That is to say, sometimes the details do not matter: certain aspects of complex behaviour transcend the particularities of a given system, and are to be anticipated in any system of a multitude of simultaneously interacting components. There can be no tougher test of this idea than that posed by the nature of human social systems. Can there really be any similarities between, say, a collection of inanimate particles in a fluid interacting via simple, mathematically defined forces of attraction and repulsion, and communities of people each of whom is governed by an unfathomable wealth of psychological complexity? The traditional approach to the social sciences has tended to view these psychological factors as irreducible components of human social interactions. But attempts to model society using the methods and tools of statistical physics have now provided ample reason to suppose that, in many situations, the behaviour of large groups of people can be understood on the basis of very simple interaction rules, so that individuals act essentially as automata responding to a few key stimuli in their environment. This is clearly a challenging, perhaps even disturbing, idea. I will review some of the evidence in its favour. I will show, with examples ranging from pedestrian dynamics to social choice theory, economics, demography, and the formation of businesses and alliances, that modelling the behaviour of individuals and social and political institutions according to the viewpoint of statistical physics does seem capable of capturing some of the important features of social systems. These models reveal many of the characteristic elements displayed by other complex systems: collective dynamics that changes via abrupt shifts (phase transitions), metastability, critical phenomena and scale-free statistical variations. I will discuss what this implies for the notions of human free will and determinism.

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Introduction

There is nothing new in the idea that human society can be analyzed using the tools and methods of physics. This belief, however alarming it might seem to some contemporary social scientists (and others), lies at the core of most liberal political theories for the past four centuries. The remarkable thing about the plethora of scientific publications today that seek to understand social phenomena using mathematical models of interacting 'particles' is not the boldness of this vision but the fact that its connection to the past has been largely forgotten. In the 17th century, the theological basis for systems of governance and social order was undermined by the emergence (one might say re-emergence; it was not unfamiliar in ancient Greece) of the concept of 'natural law', which held that human society could be understood and directed according to reason and logic. 'Everywhere', says historian George Sabine, 'the system of natural law was believed to offer the valid scientific line of approach to social disciplines and the scientific guide to social practice' [1].

But was 'natural law' really related to physics, or was it just a belief that God had made the universe an orderly place? The English philosopher Thomas Hobbes, in his treatise *De Cive* (1642), expressed no doubt that the workings of society were every bit as mechanical as the workings of clockwork:

'For as in a watch, or some such engine, the matter, figure, and motion of the wheels cannot be well known, except it be taken in sunder, and viewed in parts; so as to make a more curious search into the rights of states, and duties of subjects, it is necessary… they be so considered, as if they were dissolved, that is, that we rightly understand what the quality of human nature is, in what matter it is, in what not, fit to make up a civil government' [2].

In other words, to understand society you need to break it down into its component parts, understand their individual function, and then see how they interact with one another to generate the whole. This was the procedure recommended to natural philosophers by René Descartes in his *Discourse on Method* as a means of studying nature. Both Hobbes and Descartes were inspired by Galileo. Hobbes, who visited the famous Italian scientist in Florence in 1636, was convinced that Galilean physics established the fundamental rules governing the behaviour of the 'particles' of society: human beings.

Hobbes's analytic approach to political science was supplemented by the insistence of his friend and protégé William Petty, a founding member of the Royal Society, that social systems be studied empirically by quantifying social numbers, such as populations, budgets, trade figures and so forth [3]. Petty's approach evolved into the discipline of social statistics, from which much of the modern understanding of statistics as a branch of mathematics emerged. Scientists such as Laplace, Poisson, Maxwell and Boltzmann, as well as moral philosophers like Immanuel Kant, Auguste Comte, John Stuart Mill and Karl Marx, were influenced by the enthusiasm for a statistical perspective on social science [4]. In the light of all this, it is perhaps remarkable that it took modern statistical physics so long to begin finding applications in social science.

There are many possible reasons for that delay. One is perhaps that recent social science has tended to adopt a psychological approach to understanding human behaviour, focusing on the ways in which individuals understand and respond to their social environment. Another is no doubt the perception that social science is a 'soft' science, and therefore unsuited to (and maybe undeserving of) the rigour of the methods common to a 'hard' science like physics. Today, physicists are coming to acknowledge the truth of Herbert Simon's claim that the social and economic sciences are in fact the hardest sciences, in the sense of being the most difficult and complex, with rules that can change over time, often in an adaptive manner.

The current interest, from within the physical sciences, in so-called complex systems has also both engendered some confidence in extending their techniques to social systems and stimulated an appreciation that these systems provide a rich playground of phenomena and data within which complexity science can explore its capabilities. That is to say, social sciences are a great place to look for problems in complexity. There is also surely a profound motivation for these studies from the fact that computational methods are now able routinely to simulate systems with very many interacting components. It is possible to overstate that case, however; some of the earliest attempts to model traffic flow using physics-based models date back to the 1950s [5], and in his groundbreaking book *Micromotives and Macrobehavior* (1978), US economist Thomas Schelling [6] investigated the dynamics of simple lattice models of social behaviour by hand.

If the case is going to be made that physics can contribute to an understanding of the social sciences, that is not going to be done by any crucial experiment or theory. Rather, the argument will have to be cumulative, arising on a case-by-case basis. I shall look in the concluding section at the question of whether one can consider this argument already to be compelling. First, I shall ask what we should be looking for in seeking a physics of society. What are the characteristic phenomena of statistical physics that might be identified in social systems? I shall follow this with a description of some of the specific instances in which ideas from physics have been used both to model and to interpret social phenomena. Such examples are now rather impressive in their breadth and scope [7]. Before concluding, I will examine the question of what a physics of society might mean, and has seemed to mean in the past, for questions of free will and determinism.

The Signatures of Statistical Physics

Abrupt changes in the states of matter have been evident ever since humankind boiled their cooking pots dry or witnessed the spring thaw. The first manifestation of statistical physics – that is, the kinetic theory of gases, due largely to Clausius, Maxwell and Boltzmann – contained no prescription for these phase transitions. By modifying the theory to include intermolecular forces, however, van der Waals [8] was able to account both for the transition between a liquid and a gas and the existence of a critical point (in the phase space of temperature, pressure and density) at which this transition disappears. The former phenomenon is a first-order transi-

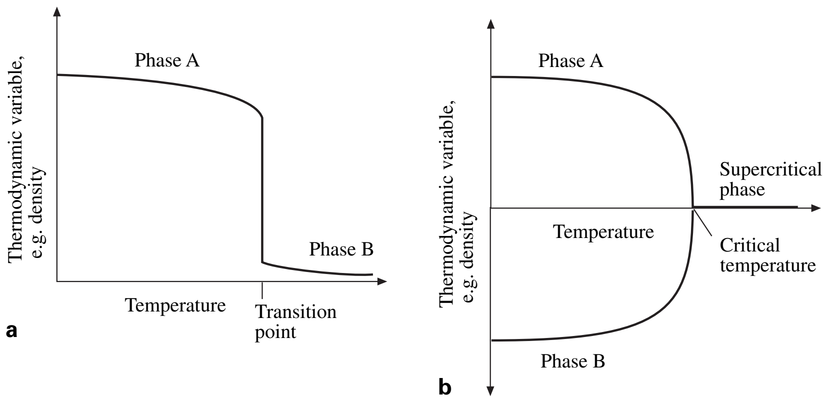

Fig. 1. First-order (**a**) and critical (**b**) phase transitions.

tion, which, loosely speaking, means that it involves an abrupt jump in a thermodynamic variable (here density). (More precisely, a first-order transition exhibits a discontinuity in the first derivative of the system's free energy as a function of some thermodynamic variable.) The liquid-gas critical point, in contrast, is a second-order phase transition (it involves a discontinuity in the second derivative of the free energy). Cooling a supercritical fluid through its critical point induces a separation into two distinct phases, liquid and gas. There is no jump, but rather a discontinuity in the slope of the density of the system. The transition involves a bifurcation, whereby a state that was initially homogeneous separates into two distinct phases (fig. 1).

Phase transitions are collective phenomena. This is not just to say that the very notion of a liquid or gas phase has no meaning for an individual molecule; rather, it implies that all the molecules in the system change their dynamical behaviour at once. At thermodynamic equilibrium, a liquid is liquid-like everywhere. Even at just a fraction of a degree above its melting point, a substance does not contain residual pockets of solid. A phase transition is an all-or-nothing affair.

The way in which these transitions happen differs if they are first-order or second-order (or more generally for the latter, 'critical', meaning that the discontinuity is in the second or higher derivative of the free energy). A first-order transition proceeds via nucleation and growth of the new phase. Precisely at the transition point – say, the melting temperature of a solid – the free energies of the two phases, liquid and solid, are equal. Slightly below the melting point, the solid has the lower free energy. But because there is an energetic cost to forming a solid-liquid interface, a very small crystal can be formed only at a cost in free energy: the free-energy benefit of solidifying a small volume of liquid is outweighed by the cost of the interface that must be formed. Because the former is proportional to the cube of the crystal size, whereas the latter scales as the square of the size, the favourable free energy on freezing comes to dominate over the surface terms as the crystal seed grows. But growth of this seed must first overcome an energy barrier due to the surface free energy. So there is a nucleation barrier to the transition, and as a result, the liquid can be supercooled into a metastable state if the nucleation rate is suppressed – for example, by removing specks of dust or surface irregularities that lower the nucleation barrier. Because, likewise, a solid can be superheated, the phase transition can show hysteresis: melting and freezing can happen at slightly different temperatures. This is a kinetic phenomenon (governed by the rate of nucleation), and so it can depend on the path along which the change happens (for example, the rate of heating or cooling). The limit of stability of a metastable state – for example, the lowest possible density of a supercooled liquid – is called a spinodal. Beyond the spinodal point, the system is no longer metastable but will decompose spontaneously into its more stable state.

In contrast, a critical transition cannot show hysteresis. As a supercritical fluid is cooled towards its critical temperature, the density fluctuations in the fluid become more pronounced: some regions look liquid-like, and some look gas-like, and both the size and the amplitude of these fluctuations increase. More specifically, the size distribution of the fluctuations approaches a power-law statistical distribution, which it attains at the critical point (fig. 2). Moreover, the correlations between the motions of the particles in the fluid be-

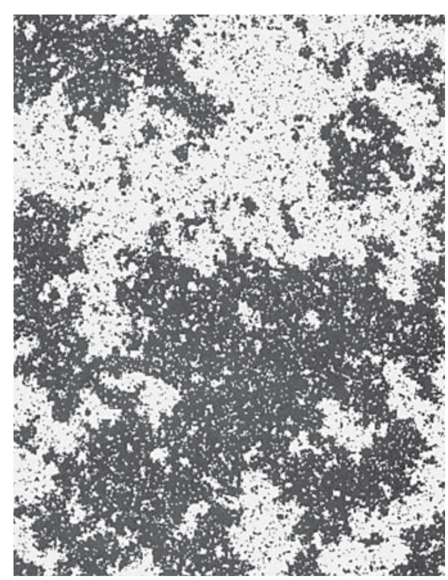

Fig. 2. Fluctuations at the critical point. In this simulation, black and white regions denote the two distinct subcritical states – for example, liquid and gas in a fluid. Patches of either phase appear on all size scales accessible to the system, and they are constantly changing in time.

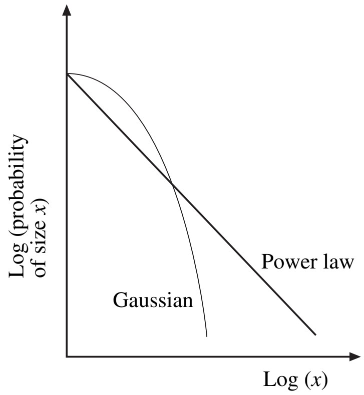

Fig. 3. Schematic of the difference between Gaussian and power-law probability distributions. Here the vertical axis shows the frequency with which some parameter has a certain size *x*, as a function of *x*. Note the logarithmic axes.

come increasingly long-ranged as the critical point is approached, and exactly at the critical point the correlation length diverges to infinity: the fluid becomes infinitely 'responsive' to perturbations, because every particle is in some sense 'in touch with' every other. This is not, of course, a reflection of any change in the range of the interparticle forces; rather, it arises because the particles become able to pass an influence from one to another across the whole range of the system (rather than that influence being eventually overwhelmed by random noise, as it is away from the critical point). If, however, the supercritical state is quenched rapidly below its critical temperature, separation into the two stable phases takes place via a process called spinodal decomposition (that is, decomposition inside the spinodal region of the phase diagram). This involves spontaneous growth of regions of the two phases throughout the system, contrasting with the nucleation and growth that occurs in a first-order transition.

Critical points are thus characterized by power-law fluctuations. In contrast to random, Gaussian fluctuations, power-law fluctuations have no characteristic size scale: they are scale-free. This means that, while the probability of a fluctuation of a certain size declines in both cases as this size increases, the decline is much slower for power-law statistics than for Gaussian statistics. To put it another way, large fluctuations are much more likely in power-law systems (fig. 3).

These, then, are some of the key motifs that could signify the operation of 'statistical physical' phenomena in a complex system of many interacting agents: abrupt phase transitions, metastability and hysteresis, collective behaviour, critical points, and power-law fluctuations or statistical behaviour. Where do we find them in society?

Crowd Behaviour

Collective motion is widespread in the animal world, and often shows some remarkable manifestations. The flocking behaviour of birds was once considered so puzzling that researchers were driven even to the extreme of postulating a kind of telepathy in order to explain how it is possible [9]. Computer simulations, however, have now shown that organisms that show this kind of swarming behaviour – not just birds, but also fish and bacteria – do not need to communicate globally in order to coordinate their actions. It is enough that they be able to respond only to the movements of their near neighbours [10–12]. A typical set of local rules that generates swarming might be as follows [10]:

Each particle responds to the movements of others within just a certain radius.

Each particle matches its speed to the average speed of others within that radius.

Each particle moves towards the centre of mass of this local group.

Each particle aims to avoid collisions with others.

Vicsek et al. [13] have shown that, in two dimensions, the effect of noise on swarming behaviour of 'self-propelled particles' is analogous to the effect of temperature on the spins of a magnetic material. In fact, this system is technically equivalent to a Heisenberg magnet, in which the spins can point in any direction in the xy plane [13, 14]. That is to say, just as heating the magnet brings about a critical phase transition from a magnetically ordered to a magneti-

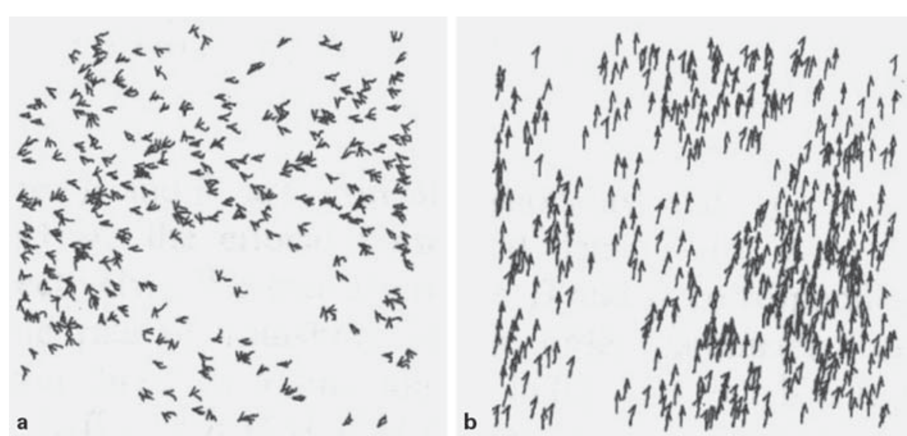

Fig. 4. A phase transition from a disordered (**a**) to an ordered (**b**) state – from random to coherent motion – takes place in a system of self-propelled particles as the amount of noise in the system decreases [from 13].

cally disordered state, so increasing noise in the system of particles (which hinders their ability to synchronize their motions) creates an abrupt transition from a swarm state, in which the particles move collectively, to a random state in which there is no coordination of the motion (fig. 4).

This suggests that there is indeed a kind of physics in animal motions. But humans are rarely in a situation where they attempt to coordinate their movements into flocking patterns. More typically, they form crowds that either have a common destination (in which case the collectivity is trivially prescribed by the boundary conditions of the system) or that lack any common purpose. In the latter case, is the result inevitably just chaos and confusion?

In 1971, Henderson [15] showed that the movements of people in crowds seem to obey the Maxwell-Boltzmann statistics of the kinetic theory of gases. Perhaps this is not so surprising; after all, these statistics are the result of so many independent, randomly moving particles. It is by no means obvious that a crowd of 'self-propelled' people should show the same distribution as a system of thermally excited particles, but neither was it clear that Henderson's statistics were really good enough to provide a stringent test of the match with the Maxwell-Boltzmann distribution. Making the connection to the kinetic theory did, however, prompt Henderson to propose that perhaps in situations that enhanced the crowd density, they might show a phase transition to a liquid-like state. I shall return to this possibility in the next section.

Helbing and Molnar [16] have modelled the movements of pedestrians more systematically using a particle-based model that enacts local rules, somewhat along the lines of the models of flocking behaviour. The model generally assumes that the walkers each have a particular destination and a particular preferred walking speed. They will maintain both their direction and their speed unless forced to slow down or deviate to avoid collision. This simple prescription is notable for its minimal assumptions about the psychology of the walkers, and yet it yields group behaviour that looks remarkably life-like. For example, walkers traversing a corridor in opposite directions will tend to form into streams [17]. There is no explicit prescription in the model for these collective modes of movement; they are an emergent property. Interestingly, they can also be considered an intelligent response to the situation, since streams help to reduce the chances of collision. This can be seen as an example of the 'wisdom of crowds' – the familiar notion that a group can arrive at a good solution to a problem even without any conscious pooling of their actions [18].

Helbing et al. [19] have seen abrupt switches in the collective motions of walkers, which have strong similarities to phase transitions. For example, when a crowd of walkers (with frictional properties when they come into contact) attempts to exit from a room through a single doorway, the rate of exit decreases rapidly above a certain threshold of the average speed with which the walkers try to move. In this 'panic' state, the walkers jam up against one another at the doorway, so that it is very hard for anyone to get out. This kind of behaviour seems to be apparent in experimental studies on mice attempting to exit from a flooded chamber [20]. Two groups of walkers passing in opposite directions down a corridor can likewise become 'jammed' if there is too much randomness in their motions – too much noise in the system. This

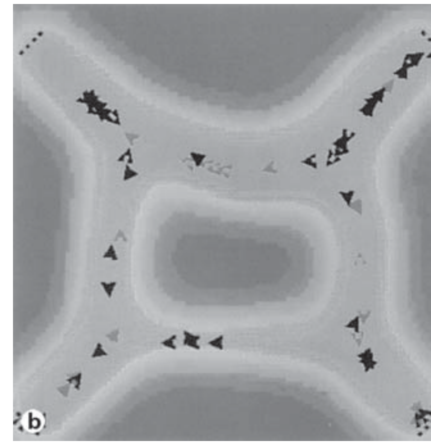

Fig. 5. The evolution of human trail systems across open spaces (**a**) can be simulated by models of pedestrian dynamics (**b**) [from 22]

looks rather like a freezing transition – albeit one that is induced in a counter-intuitive way by raising the 'temperature' (noise) in the system [21].

The 'collective' features of pedestrian motion can result in the spontaneous formation of characteristic trail patterns across empty spaces such as parks [22] (fig. 5). By developing a better understanding of these apparently 'natural' modes of human group movement, it should become possible to formulate better designs for civic and urban spaces [23] and to make more accurate predictions of the safety measures needed in buildings.

Traffic Flow

The movement of traffic is a problem of 'purposeful' granular flow very much akin to the motions of pedestrians, and it has been studied and simulated using similar particle-based models. In these, the vehicles typically aim to achieve a preferred speed and to avoid collisions. The system is rather more constrained, however, by the essentially one-dimensional nature of the motion. Both simple models [24, 25] and observations [26] suggest that there exists a threshold traffic density, above which congestion sets in rather abruptly. Thus, below the threshold the traffic moves freely – the vehicles are more or less independent of one another – and the throughput of traffic increases in proportion to the traffic density. But above the threshold, the density rises sharply and the throughput falls to close to zero, in a traffic jam (fig. 6). This looks rather like a kind of freezing transition. The free-flow state may persist metastably above the threshold density, however, until a random perturbation nucleates a jam. In both theories and observation, these jams typically propagate upstream, against the direction of traffic flow (fig. 7). A jam can sometimes increase in complexity over time, for example bifurcating into two or more distinct jams that create 'stop-and-go' waves of traffic.

Kerner and Rehborn [27] have proposed that, in addition to the free-flow and jam states of traffic, there is a third phase which they call synchronized flow. Here the traffic is dense but nevertheless continues to flow relatively smoothly because all the vehicles have synchronized their speeds and lane-changing is rare. Synchronized flow can be considered analogous to the liquid state, which intervenes between the gas-like free flow and the solid-like jams. It remains a matter of debate, however, whether synchronized flow is a fundamental traffic state or whether it is triggered only by 'inhomogeneities' such as bottlenecks and junctions [25]. Schreckenberg and colleagues [28] have argued that synchronized flow may be stabilized by a factor not taken into account in simple granular models: the drivers' desire for a smooth journey, which makes them try to avoid repeated and extreme braking and acceleration. Apart from anything else, this result supplies a reminder that simplicity is not always a virtue and that increasing the level of psychological complexity in a model can alter the global behaviour in significant ways.

The congested traffic states in these models are not always manifested in a

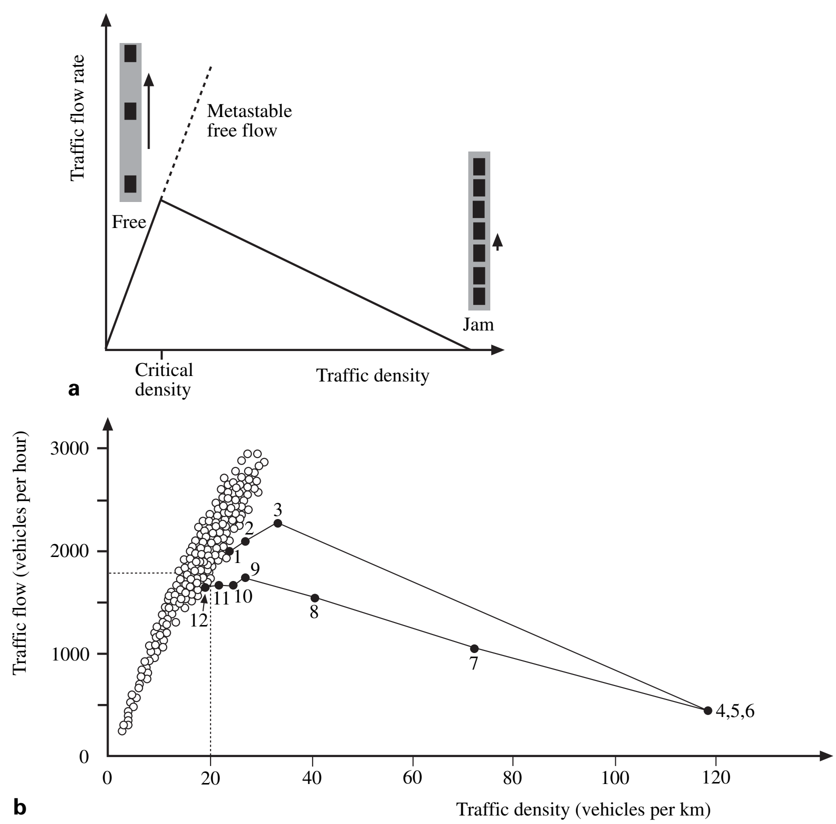

Fig. 6. The fundamental diagram of traffic flow in a simple cellular-automaton model (**a**) shows a transition from free flow to a jam state above a certain critical density, along with persistence of 'metastable' free flow above the critical density (dashed line). Some observations of real traffic flow (**b**) seem to bear out this picture. Here the numbered data points 1–12 illustrate what happens to the flow as a jam forms. They show 1-min averages over a 12-min flow sequence at one point on the highway. In other cases, however, both in theory and in the real world, the onset of congestion can be accompanied by much more complicated dynamics than this simple transition to a jam state.

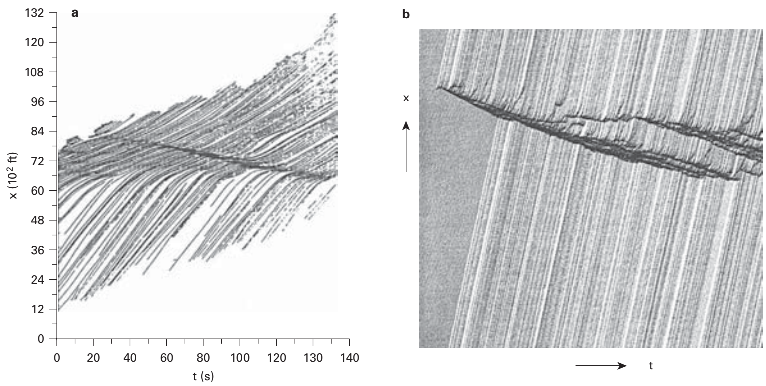

Fig. 7. Traffic jams in real data (**a**) and in a cellular-automaton model (**b**). In both cases, each line denotes a single vehicle moving along the highway: a constant slope indicates that the vehicle maintains a constant speed. When forced to slow down, the space-time line kinks towards the horizontal. In the simulations, a single perturbation – a sharply braking vehicle – in the top left creates a jam that fragments into several waves of congestion as time passes. In both cases, the jams move steadily upstream, contrary to the direction of traffic flow.

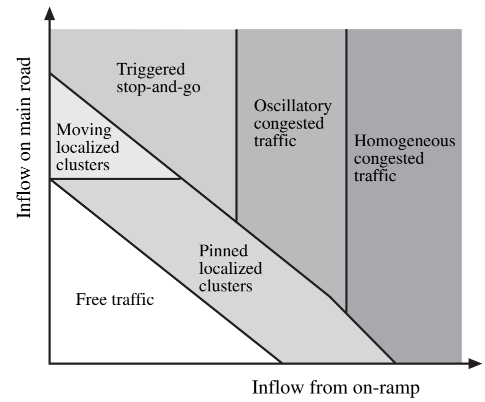

Fig. 8. The various traffic flow states that can be induced by a single perturbation (localized congestion due to an entry road or on-ramp) as a function of the flow parameters [from 29].

single, simple jam. Helbing et al. [29] have found that a perturbation such as a junction can trigger a wide range of different states, depending on the traffic density on the main highway and on the entry lane: for example, oscillatory congestion, stationary and moving jams and so forth. These changes in the nature of the flow appear quite abruptly, like phase transitions, as the flow parameters are changed, and they can be represented on a 'phase diagram' of traffic states (fig. 8).

The intersections and traffic lights of urban road networks introduce a whole new level of complexity that is, at present, far less well explored. Biham et al. [30] have found that there is an abrupt transition from free flow to total 'gridlock' on a square grid of city streets in which the flow at crossing points is governed by traffic signals. And Toledo et al. [31] find that even a single car can display chaotic dynamics when travelling through a linear set of periodically switched traffic lights with the option of speeding up to beat the lights or slowing down to anticipate them .

Quite aside from simply predicting modes of traffic flow – and particle-based simulations are already used in Germany and the USA for real-time simulation of flow on highways, pinned to data from a few automated monitoring points [32] – these studies could help to improve road design and safety measures. As vehicle technology becomes more sophisticated, for example with the addition of automated, radar-based driver-assistance systems that eliminate the problems of delay and overreaction in human responses, these models might even be coupled to control systems in individual vehicles so as to smooth out the congestion and jams that would inevitably result with imperfect human drivers at the wheel. Traffic simulations by Treiber and Helbing [33] and Davis [34] have shown that some jams in heavy traffic can be smoothed away completely if just 20% of the cars are equipped with automated systems that enable them to respond optimally to changes in traffic flow.

The Dynamics of Voting

Daily life presents us with endless choices. Indeed, many of the customs and rules of our societies have evolved largely to lighten the load of this decision making: the establishment of norms enables us to make some 'choices' without even having to think about them. Epstein has shown that a simple model of consensus-seeking agents faced with a binary choice rapidly evolves towards states in which the agents form normative clusters that require the smallest amount of opinion sampling possible: the agents find states in which most of them do not really have to 'think' about how to act [35].

But there is no escaping some choices. The classic example is the election, in which we are asked to place a vote for one candidate among several. This can be seen as an analogue of a wide range of social phenomena in which the challenge is to select one option from many. The difficulties presented by voting schemes were appreciated by Aristotle, who saw that a society divided into two political factions of com-

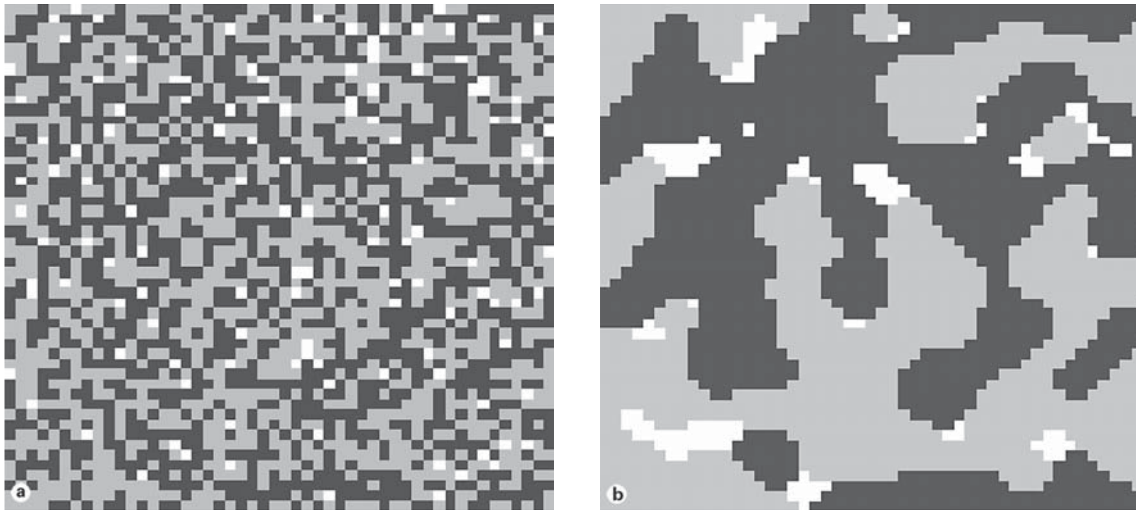

Fig. 9. In a lattice model of demographics, agents of two colours will move to a free lattice site (shown in white) if the proportion of neighbours of a different colour exceeds a certain threshold. A well-mixed initial population (**a**) evolves rapidly into one that is strongly colour-segregated (**b**) [simulations by Paul Ormerod].

parable size was inherently unstable. Condorcet [36] pointed out in the 18th century that majority rule is not always the best voting procedure, and he and others proposed various voting systems that aimed to avoid some of its shortcomings [37]. But these efforts are all undermined by Arrow's 'impossibility theorem' [38], which demonstrates that there is no democratic voting procedure that is able to meet all of the logical criteria we might demand of a 'fair' system.

Physics-based studies of voting and decision making have, however, tended to neglect such difficulties, looking instead simply at the way in which interactions affect the distribution of voting orientations in a social system. 'Orientation', a metaphor in social terms, is translated into a physical reality in these models by basing them on models of magnetism. Each voter can be represented as a magnetic atom whose spin can point in as many discrete directions as there are choices. In the simplest case of a binary decision, this can be mapped onto the two-dimensional Ising model [39–41].

Just as in the magnetic case, the orientation of one agent influences that of its neighbours: the models assume that there is a kind of social force, akin to magnetism, that encourages the 'spins' to become aligned. In other words, each agent tries to persuade its neighbours to adopt the same opinion. A simple mapping of the voting problem onto the Ising model would clearly have the rather trivial result of producing a global consensus, provided that there is not too much 'noise' in the system to allow the mutual interactions to be felt – that is, provided the system is below the critical temperature at which a switch to a randomized state takes place. This clearly does not look much like reality, in which societies tend to support a range of different opinions. But one of the key considerations is the structure of the social network through which each agent propagates its influence. Wu and Huberman [42] have shown that a voting model in which the agents act over a social network with a power-law distribution of links – the scale-free statistical form of some real social networks [43, 44] – evolves so that several groups with different opinions coexist persistently in the society. In other words, the society can sustain localized islands of opinion that do not expand to 'infect' it all.

Bernardes et al. [45] have shown that a similar model of 'magnetic' electoral voting on a scale-free network, in which the votes are to be distributed among a large number of candidates rather than just two, produces a power-law distribution of votes – which is precisely what has been observed for the 1998 Brazilian elections, in which over 100 million people voted for over 10,000 minor governmental officials [46]. The key consideration here is that these power-law statistics are *not* what would be expected if each voter were making his or her decision independently; in that case, the distribution of votes among candidates should be Gaussian. The crucial message, then, is that democratic elections do not appear to be determined by independent decisions throughout the electorate: there are strong collective effects, due to the mutual interactions that sway the opinions of voters, which skew the results away from Gaussian.

Cultural Segregation in Human Populations

This influence of interactions between agents is arguably the key factor that is explored in physics-based models of social behaviour. Social scientists have of course always recognized that individuals respond to (and influence) what others do, but they have not generally acknowledged that these collective effects can dominate the resulting social behaviour in a way that does not depend on the fine details of individual psychology. An analogy in statistical physics might be the way in which the gross behaviour of a many-particle system – the nature of its phase transitions, and in particular the scaling behaviour of properties at critical points, which define the 'universality class' of the system – may not depend at all on the detailed mathematical form assumed for the intermolecular forces. All that matters for the broad-brush collective behaviour are factors such as whether the forces are long- or short-ranged, and what the dimensionality of the system is [47].

One corollary is that individual behaviour might not be a good guide to the behaviour of a group – or conversely, the way a group acts might tell us rather little about how an individual thinks. One of the classic – and potentially controversial – demonstrations of this was provided by Schelling [6], who used a lattice model to simulate the emergence of segregation in a society.

The development of ethnic neighbourhoods in large cities is a very familiar phenomenon, and can be seen as a natural consequence of the human tendency to mix with others of the same culture, race, class or opinions. In some cases the outcome seems benign – the Chinatowns, Italian and Jewish quarters in some Western cities may become colourful tourist attractions. Elsewhere this segregation can become problematic, for example in the tensions that have smouldered between Asian and white districts in Northern British cities or in the ghettoization of black neighbourhoods following 'white flight' from US city centres.

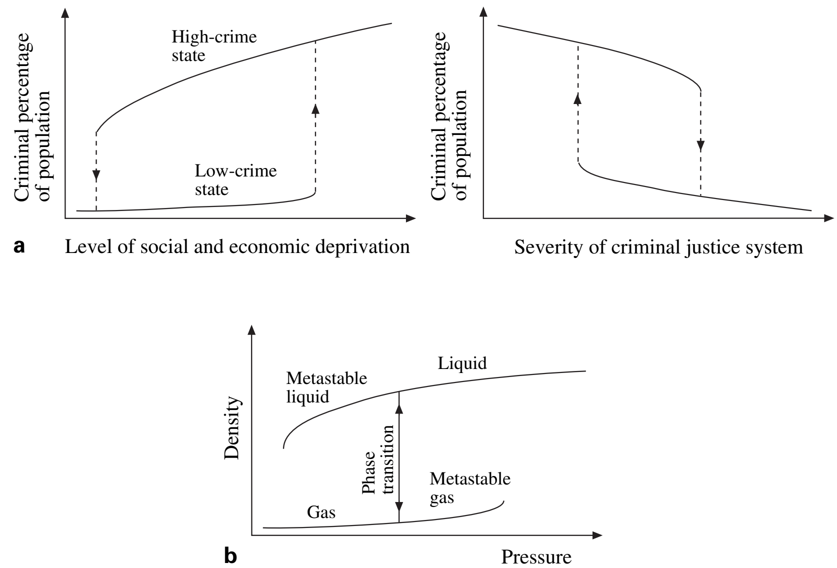

Fig. 10. The crime model of reference 49 predicts two distinct states of a society, in which there is a high and a low proportion of crime. Abrupt transitions may occur between these states as social factors, such as deprivation or the severity of the criminal justice system, are varied (**a**). These sharp jumps between two collective states are analogous to the phase transition between a liquid and a gas in the van der Waals theory of the fluid states (**b**).

A phenomenon like white flight could easily be read as an expression of substantial racial prejudice and intolerance. But Schelling [6] showed that a lattice model containing two types ('colours') of agent, in which each will move to an empty site if more than a third of its neighbours are of a different 'colour', will quickly evolve from an initially well-mixed population to one that is highly segregated (fig. 9). The segregation is motivated by a preference to be among one's 'own type', but the degree of intolerance in these agents is not especially high – they are happy to accept up to one third of their neighbours being a different 'colour'. (Indeed, segregation can develop if the threshold is even higher.) To look at it another way, one would be wrong to infer from the highly segregated society that is produced that the agents in that society must be extremely prejudiced.

This kind of 'demixing' is precisely what is seen in some two-component physical systems – for example, in polymer mixtures. The model also shows some phenomena that are readily accounted for in physical terms, such as the migration of empty spaces to the interfaces between neighbourhoods of different 'colour', like gas bubbles accumulating at interfaces to minimize the interfacial free energy.

The Spread of Crime

The spread of modes of behaviour, fashions and opinions has been perceptively discussed by Gladwell [48], who notes that it typically tends to become abrupt once it encompasses a certain fraction of a population. Gladwell calls these thresholds 'tipping points', and he compares them to the onset of an epidemic in the spread of disease. An alternative interpretation, however, can be made in terms of first-order phase transitions, in which the phase change engulfs a system once there is a critical nucleus that exceeds the nucleation threshold.

This link to phase transitions is explicit in models of the spread of social norms of crime and marriage devised by Campbell and Ormerod [49]. They make the basic assumption that crime breeds more crime – which, in an agent-based model, corresponds to the notion that individuals who are surrounded by criminals are more likely to turn to crime themselves. By the same token, individuals whose neighbours are law-abiding are more inclined to act that way too, whether out of peer pressure or because they conclude that they do not need to take the risk of committing a crime, with its attendant threat of punishment. Campbell and Ormerod do not simply divide the population into criminals and non-criminals, however. They assume that there are three groups: agents who are not susceptible to crime under any circumstances (most women and pensioners might fall into this group), active criminals, and agents who are susceptible to becoming criminals. This susceptibility is influenced by the size of the other two groups.

One of the major questions for criminologists is whether levels of crime are influenced by the severity of the penal regime. There is no consensus here; some argue for being 'tough on crime', while others say that crime is largely the result of social deprivation and that harsher sentences for criminals would have little effect. Campbell and Ormerod looked at how, in their model, changing the severity of punishment and the level of social deprivation (both of which were assumed to be linked in a straightforward way to the probability of an individual becoming a criminal) affected the level of criminality in the population.

They found that changing one of these factors while keeping the other constant resulted, in both cases, in large regimes of parameter space where two distinct states were possible: a high-crime and a low-crime society (fig. 10a). They assumed that one state would switch to another at the end of each branch. Thus, changing the social conditions could induce an abrupt improvement in or worsening of crime levels at the transition points; but elsewhere, such changes made very little difference to the state of the system. Under different circumstances, therefore, one might observe changes that either supported or denied the efficacy of being tough on crime.

These results look exactly like the phase transition between the liquid and gas states in van der Waals' model (fig. 10b). One might anticipate that the model could show a kind of 'equilibrium' transition between the two crime states somewhere around the middle of the overlap between branches, and that beyond this point the respective states become 'metastable' – and susceptible to switching if a sufficiently large region can be nucleated – until the metastability disappears at the spinodal point.

Of course, this is a very crude model. But the point is not that it is supposed to provide an accurate prediction of what might happen in a real society. Rather, it illustrates a mechanism by which collective effects convert a gradual change in 'social forces' into an abrupt switch in social behaviour. Gladwell [48] points to the 'clean-up' of New York City in the 1990s, ostensibly the result of the mayor's 'zero-

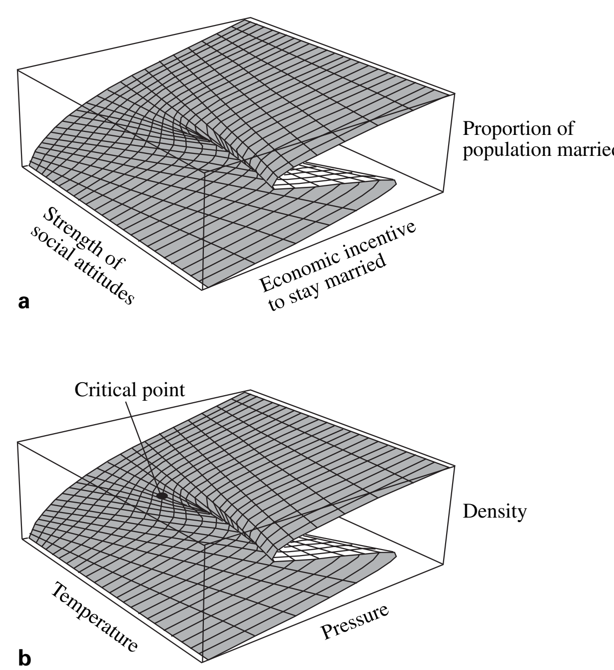

Fig. 11. A model of the effect of social factors on the proportion of married people in a society (**a**) [50] predicts a 'phase space' entirely analogous to the *P, V, T* surface of the van der Waals model (**b**).

tolerance' policy on crime, as an example. Moreover, it warns us to be wary of dogmatic assertions from politicians that a particular remedy for a social ill is always the right one, regardless of the particular social circumstances at the time.

Ormerod and Campbell [50] have developed a similar 'social pressure' model to explore the demographics of marriage, and in particular the way in which the proportion of married people depends on both the strength of social attitudes (whether marriage is 'unfashionable', or conversely whether unmarried cohabitation is condemned) and the economic incentives (such as tax advantages). They find similar two-state behaviour, this time displaying both first-order switches and a critical point – which map directly onto the *P*, *V*, *T* landscape of the liquid-gas transition in van der Waals' theory (fig. 11).

The Growth of Business Firm and Collectives

People like to form groups. They give us companionship, power in numbers, and a sense of identity and belonging. In the business world, a collective is often more effective than an individual, since, as Adam Smith [51] pointed out, the division of labour creates greater efficiencies, and many individuals can afford labour-saving machinery that lone workers could not. The seemingly inexorable need for companies to grow in order to stay competitive is believed to be driven by the concomitant increasing returns of scale, as well as by the advantages of greater market share and visibility – the 'rich-get-richer' effect.

Nonetheless, most companies go out of business. Of the 5,000 largest US firms operating in 1982, only 1,750 or so still existed in 1996. No one really knows why firms fail, although there is no lack of theories. This is not surprising, since no one really knows how firms grow either. The first attempt to formulate a theory of firm growth, by Gibrat [52] in 1931, still provides a kind of benchmark for theories today, but it does not seem to supply an accurate explanation of the observed data on firm growth and size [53]. What we *do* know is that there are many more small firms than large ones, and that their distribution seems to follow a power law [54] (fig. 12a). So, in fact, do the rates of firm growth – both positive and negative [55] (fig. 12b). These power laws suggest that, as we might expect, the growth of firms is a highly correlated phenomenon: the fate of any one firm depends strongly on what happens to the others.

Axtell [56] has devised an agent-based model of firm growth that, unlike most economic models, does not specify an increasing return of scale at the outset. The model allows for such a thing to evolve, but does not prescribe it. Firms arise from the cooperation of agents who seek to optimize their 'utility', expressed as a compromise between the amount of work they do (which generates money) and the amount of leisure time they retain. In balancing these two things, each agent has different preferences: the population is heterogeneous. Axtell finds that the model never settles into an equilibrium state – which is to say, it has no Nash equilibrium (a state in which no agent can improve its lot by changing its position). There is a constant turnover of firms of all sizes (fig. 13a). The model generates power laws both for firm sizes and firm growth rates (fig. 13b, c), as observed in reality. And it gives some insight into what makes a firm successful, suggesting that far more important than the ability to maximize profit or overall utility is the ability to acquire and retain productive workers. Firms fail when they become dominated by 'slackers' that do little work but seek to benefit from the efforts of their colleagues.

The Formation and Stability of International Alliances

The business market is commonly modelled as a competitive, Darwinian world. But sometimes even the largest firms are forced to forgo competition for collaboration. For example, all businesses suffer if a new technology lacks a technical standard – if personal computers all used different sizes of compact disks, for instance. The need for standardization has become very clear with the growth of information technology, but it has in fact always been an aspect of commerce and technology, from the width of railway gauges to the competition between imperial and metric units.

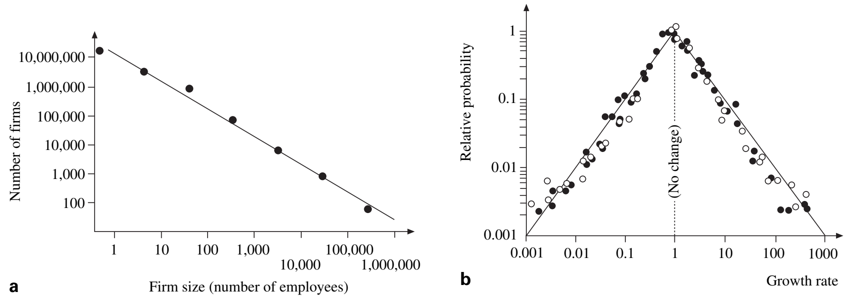

Fig.12. Both the distribution of firm sizes in the US (**a**) [54], and their rates of growth (**b**) [55] follow power laws. In **b**, black circles show sales, and white circles show number of employees: both are measures of a firm's 'size'.

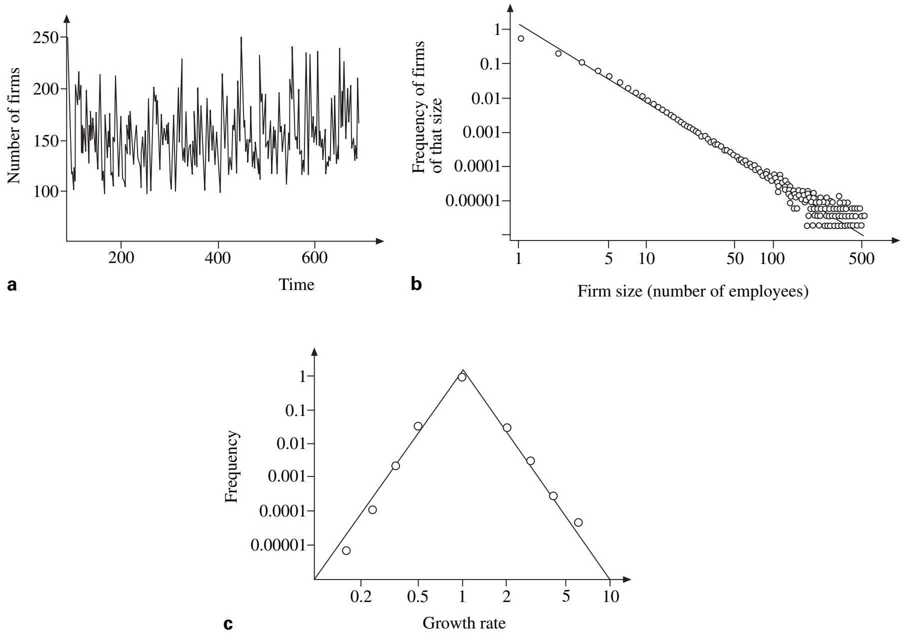

Fig.13. Axtell's model of firm growth [56] has no stable equilibrium state – there is a constant turnover of firms (**a**). The model generates the power laws in firm size (**b**) and growth rate (**c**) seen in reality (fig. 12).

Typically, rival technical standards might emerge from two competing camps, so that all businesses in an industry will be compelled to join one alliance or another, none of them being big enough to secure the market alone. Axelrod and Bennett [57] have developed a model for the formation of alliances which they have applied to the case of standardization of the Unix computer operating system in the 1980s [58]. The model treats the agents (in this case, the businesses) as particles that experience varying degrees of attraction and repulsion for one another, depending on factors such as the company size and the degree of overlap of their product ranges. The most stable configuration of allied companies is found by minimizing the 'energy' of this many-particle system, based on these 'individualized' interparticle forces. They found that, if the number of alliances was restricted to just two, the model predicted a division of the nine computer firms involved into alliances very close to those that developed in reality. (If the number of alliances is unconstrained, however, the energy landscape is much rougher, and no clear outcome is predicted above others [59].)

This 'landscape model' has also been used by Axelrod and Bennett [57] to predict the formation of political alliances between the 17 European nations near the onset of the Second World War. This time the forces between particles (nations) are estimated according to a rather more complex set of criteria, taking into account six factors that could determine the degree of friendship or antagonism. This assignation of what Tolstoy termed 'the force that moves nations' [60] is clearly very crude and approximate; but nonetheless, the landscape model reproduced the Axis and Allied coalitions (or rather, something very close to it) for a wide range of plausible parameters, according to the political situation in 1936. The predictions fall exactly into line with the historical outcome if 1939 data are used to calculate the forces, rather than 1936 data. (This analysis too has been criticized for artificially constraining the particles to form just two 'clusters' [59]; but it has been argued, and history seems to bear this out, that international war generally imposes such a constraint [61].)

Again, one should not take the quantitative aspects of these models too seriously. What they do suggest is that considering social 'actors' (be they individuals, firms, institutions or nations) as particles that interact via forces of attraction and repulsion, seeking states that are in some sense 'energetically favourable', does seem at least to be a perspective worth investigating. If this kind of approach were to gain further support, it raises the intriguing idea that history itself might be sketched out as a landscape of possibilities with associated, crudely quantifiable probabilities – a new way of exploring what historians call 'counterfactual history' [62].

The Transmission of Cultural Traits

Ideas and opinions pass not just from person to person but from culture to culture. Historically this has often been seen as a good thing: the Arabic culture of the early Middle Ages assimilated the ancient Greek learning that it found in Alexandria, for instance, and thereby kept it alive until transmitting it to the West via Spain in the 11th and 12th centuries. On the other hand, today there is much concern about whether local customs, languages, traits, food and so forth can survive English-speaking (mostly American) hegemony that threatens to turn the world into a monoculture. But it remains unclear just how cultural traits are spread – why, for example, a language like Basque can survive for centuries amid a Romance-based culture, while languages like Manx have come close to extinction.

When do groups adopt a common culture, and when do they retain their differences? Answering that question could provide insights into a variety of important international and social processes, such as the formation of nation states, the reasons for wars of succession, the stability of transnational institutions such as trade agreements or international courts, and the spread of globalization. The tragic civil conflicts that erupted in the Balkans and in Africa during the 1990s are a reminder that cultural differences can persist beneath the façade of apparently unification, while the recent expansion of the European Union raises questions about how much difference can be accommodated within a common set of goals.

It has been suggested that a precondition of cultural convergence is some existing similarity: 'the transfer of ideas occurs most frequently between individuals… who are similar in certain attributes such as beliefs, education, social status, and the like' [63]. Axelrod [64] has used this principle to devise a lattice model of the transmission of cultural values. He assumes that a culture can be characterized by a list of features or 'dimensions': these might, for example, include language, religion, dress, food and so forth. All cultures are assumed to display these features, but they generally differ from one culture to another: they all have a language, for example, but not necessarily the same one. So each feature in Axelrod's cultural landscape can have one of several alternative 'values', called traits, denoted simply by a number. Thus, for example, there might be five different languages in the model: five different values (1–5) of the 'language feature'. (There is no quantitative significance to these values: they are just labels.)

Each culture, then, is denoted by a certain number of features, each of which has a distinct value. The landscape is divided into cells, each representing a cultural unit – a village, say. Part of such a grid, in which there are five cultural features that each have ten different traits (0–9), is depicted in figure 14. The grid evolves by picking two neighbouring cells at random and seeing how much cultural similarity they have: how many features have the same value. With a probability proportional to this degree of similarity, the two sites then interact by setting the value of a randomly selected feature on the chosen site equal to that on its neighbouring site.

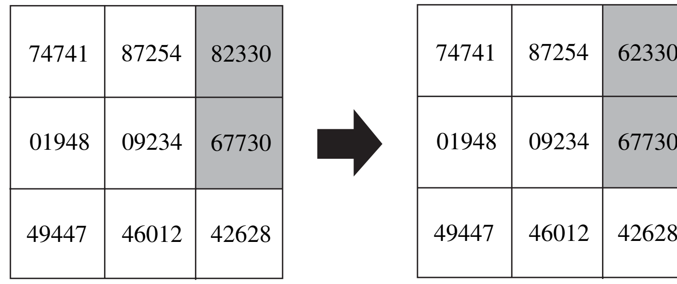

Fig. 14. One 'step' in the dissemination of culture in Axelrod's lattice model [64], for a system in which the cultures have five features each with ten distinct traits. The cultures chosen for comparison, leading to convergence, are highlighted in grey.

Clearly, this process induces convergence between the initially dissimilar cultures on the grid. But how far does this convergence proceed? Two neighbouring sites can interact only if they share at least one trait in common. So either the entire grid could evolve into a monoculture, or it could become frozen into a discrete number of non-interacting 'island' cultures (fig. 15a). It is not intuitively obvious what the result will be, nor how this will depend on factors such as the grid size (or geometry), the number of features, and the number of traits for each feature. Axelrod finds that the number of stable regions in the final state decreases (rapidly) with an increasing number of features per site, increases with an increasing number of traits per feature, and at first increases and then decreases with an increasing grid size.

The first two observations can be understood on the basis that both increasing the number of features and decreasing the number of traits increases the probability of similarities, and thus convergence, between neighbouring sites. The influence of grid size is the result of two conflicting factors. A very small grid cannot support much cultural diversity because of sheer spatial limitations on diversity – the well-known 'island effect' that restricts the number of evolutionary niches for biological diversity on small islands. On a very large grid, on the other hand, interactions can be sustained for longer (because there are simply more sites), so inert configurations of island cultures are less easily frozen in.

Castellano et al. [65] have investigated in more detail the effect of increasing number of traits on the cultural diversity (characterized by the size of the largest domain). They find that the switch between a monoculture – where the largest domain encompasses more or less the whole grid – and a fragmented multicultural patchwork happens rather abruptly as the number of traits is increased (fig. 15b). This changeover has the characteristics of a phase transition, which can either be first-order or critical, depending on the model parameters. In either case, however, the size distribution of the cultural regions at the transition point has a power-law form. The transition becomes increasingly sharp as the size of the grid increases, comparable to the way that a finite system size induces 'rounding' or blurring of phase transitions such as melting in physical systems.

Economic Markets

One of the best-studied examples of many-body social systems from a physics-based point of view is the behaviour of economic markets [66–68]. The literature in this area has already grown so vast that I cannot possibly do it justice here. I shall simply point out that economic systems seem to show many of the phenomena I have already discussed: power-law statistical distributions (for example, in the fluctuations of stock values [69]), collective behaviour and abrupt, global changes (such as the herding effect on the trading floor [68, 70]), relative indifference to detailed psychological assumptions about traders (indeed, certain aspects of market dynamics can be reproduced by assuming that the traders show 'zero intelligence', acting at random [71]), and critical points [67]. It seems clear that these studies can potentially extend classical economic theory in useful ways, for example by including trader interactions and interdependence directly (rather than indirectly via their effect on prices), allowing for heterogeneity and irrationality in trading practices, moving beyond incorrect assumptions of Gaussian statistics, and treating the economy as a truly non-equilibrium system.

Free Will and Determinism

When statistics were first introduced into studies of social behaviour in the 18th century, some commentators reacted with dismay. Was this not subjecting human nature to a tyranny of numbers? Immanuel Kant was ruthlessly unsentimental about it, however:

'Whatever concept one may hold, from a metaphysical point of view, concerning the freedom of the will, certainly its appearances, which are human actions, like every other natural event are determined by universal laws' [72].

There is some careful hedging here: Kant does not say that there is no such thing as free will, but only that it does not matter very much: the outcome still has the mathematical regularity and predictability of a 'natural law'. He went on to be more explicit about this:

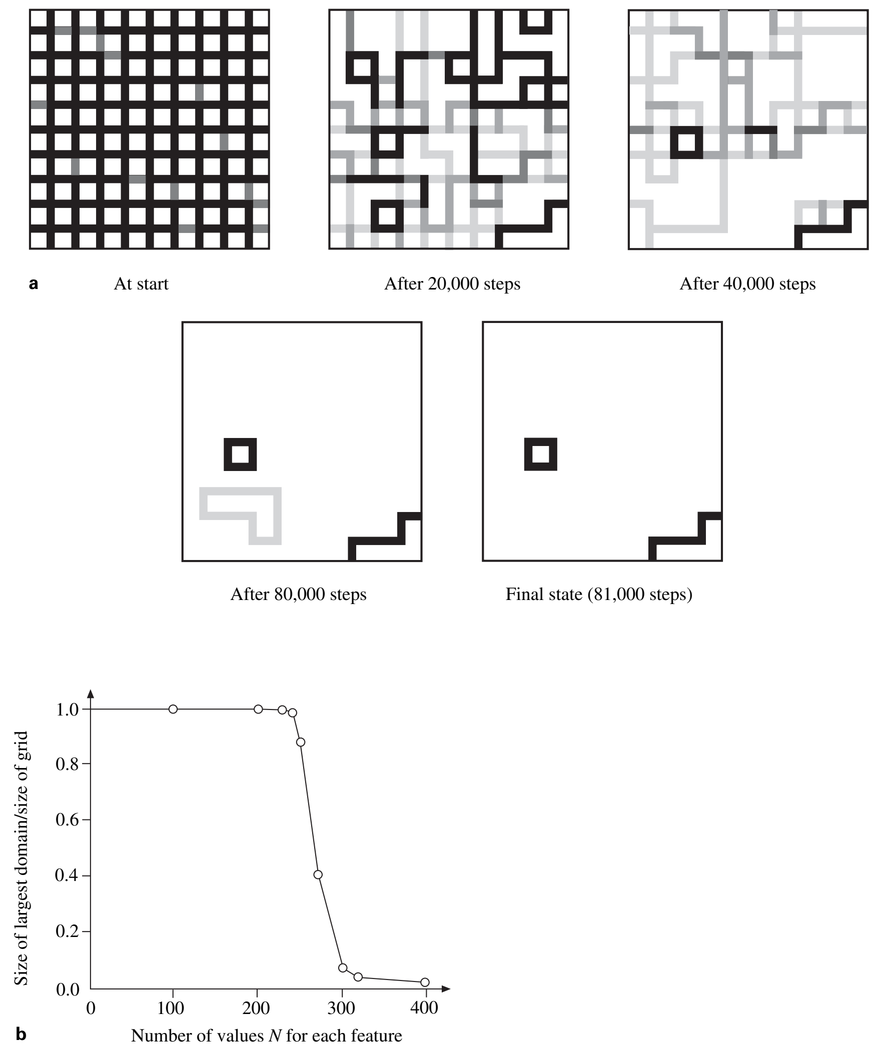

Fig. 15. Axelrod's model typically evolves from a grid of 'island cultures' to a state in which one island grows to dominate the grid (**a**). Here, the dark lines show the boundaries between grid cells, and are shaded according to the degree of similarity between adjacent cultures. The final state in this run has three distinct cultures that can no longer interact [from 64]. The geographical extent of the largest culture in the final state depends on how many values N each cultural feature can take. As N increases, there is an abrupt switch from an essentially monocultural state to a grid with a lot of cultural diversity (**b**). Here this switch happens in a phase transition at around $N = 240$ [from 65].

'Individual men, and even whole nations, little think, while they are pursuing their own purposes… that they are advancing unconsciously under the guidance of a purpose of nature which is unknown to them' [72].

Others were careful to spell out that these 'universal laws' were purely statistical: they were apparent only when large numbers of individuals were taken into account. In *A System of Logic* (1862), John Stuart Mill said:

'Very events which in their own nature appear most capricious and uncertain, and which in any individual case no attainable degree of knowledge would enable us to foresee, occur, when considerable numbers are taken into the account, with a degree of regularity approaching to mathematical' [73].

That would seem to be a reasonable statement of what is being suggested in the models I have discussed above. But some 19th century statisticians sailed closer to the wind: the arch-enthusiast for 'social mechanics', the Belgian Adolphe Quételet, felt that the emergence of social averages carried a moral aspect, so that it was good to aspire towards the mean:

'An individual who epitomized in himself, at a given time, all the qualities of the average man, would represent at once all the greatness, beauty and goodness of that being…Deviations more or less great from the mean have constituted…ugliness in body as well as vice in morals and a state of sickness with regard to the constitution' [74, 75].

This seems to be an endorsement of social conformity and uniformity – a prescription for a bland, colourless society. No wonder, then, that Fyodor Dostoevsky responded to this passion for making society statistical and mathematical with an assertion that it implied the end of free will, and by implication, the end of humanity:

'As a matter of fact, if ever there shall be discovered a formula which shall exactly express our wills and whims; if there ever shall be discovered a formula which shall make it absolutely clear what those wills depend upon, and what laws they are governed by, and what means of diffusion they possess, and what tendencies they follow under given circumstances; if ever there shall be discovered a formula which shall be mathematical in its precision, well, gentlemen, whenever such a formula shall be found, man will have ceased to have a will of his own – he will have ceased even to exist' [76].

Should we be prepared, in developing a physics of society, for objections like Dostoevsky's? Probably – but there are several ways of responding to them. First, the fact is that the kind of models I have discussed are, if anything, a way of putting choice back into the kind of social modelling that, based on the economic notion of rational maximizers, threatens to portray people as automata. There is not always a single, rational 'best choice' in a given situation; or even if there is, people do not always take it. These physics-based models explicitly allow for a multiplicity of choices, which are typically assumed to be made not on rigidly deterministic grounds but on a probabilistic basis. All they really assert is that those choices are rarely independent: we are influenced by what others choose. Once that happens, the outcome can be non-linear and non-intuitive.

Second, it is fair to say that free will is over-rated. To most people, it implies the exercising of a completely free choice. But our choices are rarely free. They are influenced by all manner of considerations – not just what others do (or prevailing social norms), but by our own immediate circumstances, by advertising, by our social backgrounds and so forth. Some of these factors are surely unquantifiable – but this does not mean we cannot at least gauge the tendencies they induce. Politicians and advertisers, of all people, know how to be persuasive: if they could not influence decision making with at least some degree of calculated effect, they would not survive for long.

What is more, free will is typically severely constrained by the social setting. My free will as a driver does not lead me to drive on the wrong side of the road. My free will as a consumer does not persuade me to go shopping naked, or to buy one hundred loaves of bread. My free will as a voter does not lead me to vote for my mother as prime minister. Physics-based models of social behaviour are often tractable not because behaviours are prescribed but because the range of options is so limited. This is why it is most unlikely that anyone will ever develop a physics-based theory of how to write a novel. Social physics needs to cultivate an attitude of humility, but so should advocates of free will.

How Seriously Should We Take It?

For the physicist, the more immediate question is perhaps: is this really physics at all? At least, that is often the way the question is phrased by sceptics. I think it is more pertinent to ask: is it really social science? The models are often very precisely defined, to the extent that, for example, one can say for sure that a sudden change is a first-order phase transition, and not just something that looks a bit like it. Analogies with familiar physical systems such as the Ising model and order-disorder transitions can be made formally exact, not just loose comparisons [13, 14]. But does any of this come close to capturing real social behaviour?

I think it is fair to say that at present that is largely an unresolved issue. One reason for this is that the data necessary to put the models to the test are often largely absent. Collecting social statistics is an immense and difficult task, and it should not be forgotten that it also requires great skill and experience of the sort that social scientists have and physical scientists do not. Many social phenomena are one-off experiments, without controls and without the option of altering crucial parameters independently. It is sometimes not even clear what the relevant parameters are: that is to say, how the parameters of a particular idealized model translate into anything resembling the real world. How do you enumerate the features that characterize a society, and how do you count up the respective traits? Is it truly meaningful to dissect a culture in this way?

This is why those social systems that have perhaps yielded the best examples for physics-based modelling are highly constrained ones, such as road traffic. There are also a few social systems, such as the economic markets, for which the sheer volume of readily available and clear-cut data makes some progress possible. But these instances are rather rare, and I suggest that it is wisest at this stage to regard many studies of the 'physics of society' as doing no more than providing 'toy models' that have to be taken with a pinch of salt.

On the other hand, this does not mean that such studies have no value. On the contrary, I would argue that their primary value is often to challenge entrenched preconceptions about how human society works. Policy makers are all too prone to linear thinking: they assume that if we understand how an individual tends to think or behave, we can understand what a population will do. It is surely time to move beyond this 'ideal gas' position and to acknowledge that the interactive nature of society makes it a truly complex and non-linear system. Physics-based modelling tells us not only that interactions may change the picture entirely, relative to a linear extrapolation from individuals. It also shows that this injection of complexity does not necessarily make the problem impossible, for there are likely to be robust modes of collective behaviour that remain relatively insensitive to the fine details and idiosyncracies of individual actions and responses. And that seems worth knowing.

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